Design of fixed bed adsorption columns

Fixed bed is a widely used method for adsorption of solutes from liquid or gases. Granular particles are packed inside the fixed-bed. The fluid to be treated is passed down through the packed bed at a constant flow rate. Mass transfer resistances are important in the fixed bed process and the process is unsteady state. The overall dynamics of the system determines the efficiency of the process rather than just the equilibrium considerations.

Adsorbate concentration in solution varies over the length of the reactor and as a function of time.
Breakthrough concentration curve

As seen in the figure below, the major part of the adsorption at any time takes place in a relatively narrow adsorption or mass-transfer zone. As the solution continues to flow, this mass transfer zone, which is S-shaped, moves down the column. The outlet concentration remains near zero until the mass-transfer zone starts to reach the tower outlet at time $t_4$. Then the outlet concentration starts to rise and at $t_5$ the outlet concentration has risen to $c_b$, which is called the break point.

After the break-point time is reached, the concentration $c$ rises rapidly up to point $c_d$, which is the end of the breakthrough curve where the bed is considered ineffective. The break-point concentration represents the maximum that can be discarded and is often take as 0.01-0.05 for $c_b/c_0$. The value $c_d/c_0$ is taken as the point where $c_d$ is approximately equal to $c_0$. 
FIGURE 12.3-1. Concentration profiles for adsorption in a fixed bed: (a) profiles at various positions and times in the bed, (b) breakthrough concentration profile in the fluid at outlet of bed.
FIGURE 9-18
Carbon adsorption breakthrough curve showing movement of an adsorption zone.43
Capacity of column and scale-up design method

A narrow mass-transfer zone makes efficient use of the adsorbent. The mass-transfer zone width and shape depends on the adsorption isotherm, flow rate, mass-transfer rate to the particles, and diffusion in the pores. Experiments in laboratory scale are needed in order to scale up the results.


FIGURE  Schematic diagram of composite adsorbent pellet showing the three principal resistances to mass transfer.
If the entire bed comes to equilibrium with the feed, the total or stoichiometric capacity of the packed-bed tower is proportional to the area between the curve and a line at \( c/c_0 = 1 \). The total shaded area represents the total capacity of the bed as follows:

\[
t_t = \int_0^\infty \left( 1 - \frac{c}{c_0} \right) \, dt
\]

where \( t_t \) is the time equivalent to the total capacity. The usable capacity of the bed up to the break-point time \( t_b \) is the crosshatched area.

\[
t_u = \int_0^{t_b} \left( 1 - \frac{c}{c_0} \right) \, dt
\]

where \( t_u \) is the time equivalent to the usable capacity or the time at which effluent concentration reaches its maximum permissible level. \( t_u \) is usually very close to \( t_b \).

\[\text{FIGURE 12.3-2. Determination of capacity of column from breakthrough curve.}\]
The ratio $t_u/t_t$ is the fraction of the total bed capacity or length utilized up to the break point. Hence, for a total bed length of $H_T$, $H_B$ is the length of bed used up to the break point,

$$H_B = \frac{t_u}{t_t} H_T \quad (18)$$

The length of unused bed $H_{UNB}$ is then the unused fraction times the total length.

$$H_{UNB} = \left(1 - \frac{t_u}{t_t}\right) H_T \quad (19)$$

The $H_{UNB}$ represents the mass-transfer section or zone. It depends on the fluid velocity and is essentially independent of total length of the column. Therefore, $H_{UNB}$ can be measured at the design velocity in a small column packed with the desired adsorbent. Then the full-scale adsorber can be designed by first calculating the usable bed length up to the break point ($H_B$) which is proportional to $t_b$. Finally the length of the mass-
transfer section, $H_{UNB}$, is added to the length needed, $H_B$, to obtain the total length $H_T$.

$$H_T = H_B + H_{UNB}$$  \hspace{1cm} (17)

A simple procedure is to assume that the breakthrough curve is symmetrical at $c/c_0 = 0.5$ and $t_s$. Then $t_t = t_s$. This means that the area below the curve between $t_b$ and $t_s$ is equal to the area above the curve between $t_s$ and $t_d$.

**Example. Scale-up of laboratory adsorption column.**

A waste stream of alcohol vapor in air from a process was adsorbed by activated carbon particles in a packed bed having a diameter of 4 cm and length of 14 cm containing 79.2 g of carbon. The inlet gas stream having a concentration $c_o$ of 600 ppm and a density of 0.00115 g/cm$^3$ enter the bed at a flow rate of 754
cm³/s. The concentrations of the breakthrough curve are given below.

<table>
<thead>
<tr>
<th>Time (h)</th>
<th>0</th>
<th>3</th>
<th>3.5</th>
<th>4</th>
<th>4.5</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>c/c₀</td>
<td>0</td>
<td>0</td>
<td>0.002</td>
<td>0.03</td>
<td>0.155</td>
<td>0.396</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time (h)</th>
<th>5.5</th>
<th>6</th>
<th>6.2</th>
<th>6.5</th>
<th>6.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>c/c₀</td>
<td>0.658</td>
<td>0.903</td>
<td>0.933</td>
<td>0.975</td>
<td>0.993</td>
</tr>
</tbody>
</table>

Set the break-point concentration as c/c₀ = 0.01,

(a) Determine the break-point time, the fraction of total capacity used up to the break point, and the length of the unused bed. Also determine the saturation loading capacity of the carbon.

(b) If the break-point time required for a new column is 6.0 h, what is the new total length of the column required?
Solution:

(a) for \( c/c_0 = 0.01 \), the break-point time is \( t_b = 3.65 \) h from the graph, \( t_d = 6.95 \) h. The time equivalent to the total capacity of the bed is

\[
t_t = \int_0^\infty \left( 1 - \frac{c}{c_0} \right) dt = A_1 + A_2 = 3.65 + 1.51 = 5.16 \text{ (h)}
\]

The time equivalent to the usable capacity of the bed up to the break-point time is

\[
t_u = \int_0^{t_b} \left( 1 - \frac{c}{c_0} \right) dt = A_1 = 3.65 \text{ (h)}
\]

Hence, the fraction of total capacity used up to the break-point is \( t_u/t_t = 3.65/5.16 = 0.707 \).
The length of the used bed is $H_B = 0.707(14) = 9.9$ cm. The length of unused bed is

$$H_{UNB} = \left(1 - \frac{t_u}{t_t}\right)H_T = (1 - 0.707)(14) = 4.1 \text{ (cm)}$$

To determine the saturation capacity of the carbon, air flow rate is

$$Q = (754 \text{ cm}^3/\text{s})(3600 \text{ s/h})(0.00115 \frac{\text{g}}{\text{cm}^3})$$

$$= 3122 \text{ (g air/h)}$$

Total alcohol adsorbed is

$$\left(\frac{600 \text{ g alcohol}}{10^6 \text{ g air}}\right)(3122 \text{ g air/h})(5.16 \text{ h}) = 9.67 \text{ g alcohol}$$

Saturation capacity (Equilibrium isotherm at $c=600$ ppm) is

$$q = 9.67 \frac{\text{g alcohol}}{79.2 \text{ g carbon}}$$

$$= 0.122 \frac{\text{g alcohol}}{\text{g carbon}}$$
(b) For a new $t_b$ of 6.0 h, the new $H_B$ is obtained simply from the ratio of the break-point times multiplied by the old $H_B$.

$$H_B = \frac{6.0}{3.65} (9.9) = 16.3 \text{ (cm)}$$

$$H_T = H_B + H_{UNB} = 16.3 + 4.1 = 20.4 \text{ (cm)}$$

The fraction of the new bed used up to the break point is now

$$H_B / H_T = 16.3 / 20.4 = 0.799$$
**Bohart-Adams model (Bed Depth-Service Time, BDST)**

This model assumes that the adsorption rate is proportional to both the residual adsorbent capacity and the remaining adsorbate concentration. The bed service time is

\[ t = aD + b = \frac{N_0}{c_0 V} D \left( 1 - \frac{1}{K c_0} \ln \left( \frac{c_0}{c_B} - 1 \right) \right) \]

where

- \( V \) = superficial velocity (\( \text{ft}^3 \) liquid/\( \text{ft}^2 \) adsorbent/h)
- \( D \) = depth of adsorber bed (ft)
- \( K \) = rate constant (\( \text{ft}^3 \) liquid/lb solute/h)
- \( N_0 \) = adsorptive capacity (lb solute/\( \text{ft}^3 \) adsorbent)
- \( c_0 \) = influent solute concentration (lb solute/\( \text{ft}^3 \) liquid)
- \( c_B \) = solute concentration at breakthrough.

The critical bed depth (\( D_0 \)) is defined as the theoretical depth of adsorbent that is sufficient to prevent the effluent solute concentration to exceed \( c_B \) at zero time, which is calculated from the BDST equation when \( t=0 \):

\[ D_0 = \frac{V}{K N_0} \ln \left( \frac{c_0}{c_B} - 1 \right) \]
The adsorptive capacity of the system, $N_0$, and the rate constant, $K$, can be evaluated from the slope and intercept of the plot of $t$ versus $D$, which yields a straight line. The slope term, $a$, provides a measure of the velocity of the adsorption zone (the speed at which carbon is exhausted). The velocity of the adsorption zone is $1/a = (c_0 V)/N_0$.

The rate of carbon utilization

$$= (\text{area of column}) \left( \frac{1}{a} \right)(\text{bed density of carbon})$$
The horizontal distance between the curve representing exhaustion and breakthrough represents the height of an adsorption zone.
When a number of columns in series are used, the total number of columns is

\[ n = (\text{AZ} / \text{d}) + 1 \]

where \( n \) = number of columns in series
(rounded up to a whole number)

\( \text{AZ} \) = height of an adsorption zone

\( = \text{MTZ} \) = mass transfer zone

\( \text{d} \) = height of a single column

**Example. Preliminary design of carbon adsorption system.** A ground water flowing at a rate of 0.145 m\(^3\)/min (55,000 gal/day) requires treatment to reduce organic concentration from 89 mg/L to 8.9 mg/L (90% removal). Laboratory studies are run in 2.3 m (7.5 ft) long by 0.051 m (2 in) diameter columns at a flow rate of 0.5 L/min (190.2 gal/day). The results are shown in the previous two figures. Assume a unit weight of carbon of 481 kg/M\(^3\) (30 lb/ft\(^3\)).

Determine: The height of the adsorption zone, number and size of columns required, and the carbon usage rate.
Solution.
1. Height of adsorption zone, AZ = 2.5 m (8.25 ft)
   (from Fig. 2)
2. Number and size of units:
   \[ n = \frac{AZ}{d} + 1 = \frac{2.5}{2.3} + 1 = 2.09; \]
   therefore, number of units = 3 columns.
Area of lab columns = \( 2.043 \times 10^{-3} \text{ m}^2 \) (3.14 in^2)
Loading rate in laboratory columns:
\[ V = \frac{5.0 \times 10^{-4} \text{ m}^3/\text{min}}{2.043 \times 10^{-3} \text{ m}^2} \]
= 0.245 m^3/m^2.min (6.14 gal/ft^2.min).
Using the same loading rate for the full scale unit yields:
\[ \text{Area} = \frac{Q}{V} = \frac{0.145 \text{ m}^3/\text{min}}{0.245 \text{ m}^3/\text{m}^2\text{min}} = 0.59 \text{ m}^2 \)
Diameter = 0.867 m (2.84 ft)

3. BDST equation for 90% removal (use curve for 10% of feed concentration in Fig. 1)
Slope \( a = 5.8 \text{ days/0.91 m} = 6.34 \text{ days/m} \) (1.93 days/ft)
Intercept \( b: -7 \text{ days} \)
Equation of line: \( t = 6.34X - 7 \)
Velocity of adsorption zone = \( 1/a = 0.158 \text{ m/day} \) (0.52 ft/day)
Carbon utilization = Area x (1/a) x Unit weight
\[ = 0.59 \text{ m}^2 \times 0.158 \text{ m/day} \times 481 \text{ kg/M}^3 \]
\[ = 44.8 \text{ kg/day} \) (131 lb/day)