Multicomponent Distillation

- need more than one distillation tower, for \( n \) components, \( n-1 \) fractionators are required
- Specification Limitations

The following are established at the beginning:
1. Temperature, pressure, composition, and rate of the feed.
2. Pressure of the distillation
3. Feed to be introduced on that tray which will result in the least total number of trays (optimum feed-tray location)
4. Heat loses (even if assumed to be zero)

Under these circumstances only three additional items of the following list need to be specified, any other items must be calculated.

1. Total number of trays.
2. Reflux ratio
3. Reboil ratio (ratio of vapor produced by the reboiler to residue withdrawn)
4. Concentration of one component in one product (a maximum of two may be chosen)
5. Ratio of flow of one component in the distillate to the flow of the same component in the residue, or split of the component (a maximum of two may be chosen)
6. Ratio of total distillate to total residue

Key components

The feed components are listed in order of their relative volatility. The more volatile components are called light, the less volatile are heavy. The components separated in a tower are called light key (more volatile) and heavy key (less volatile). Any components lighter than the light key are called light components, while those heavier than the heavy key are called heavy components.

In hydrocarbon systems, because of nonidealities, the equilibrium data are often represented by
\[ y_A = K_A x_A, \quad y_B = K_B x_B, \quad y_C = K_C x_C, \quad y_D = K_D x_D \]
where \( K_A \) is the vapor-liquid equilibrium constant or distribution coefficient for component A. K is a function of temperature and pressure, but is assumed to be independent of composition, which is sufficiently accurate for most engineering calculations. For an ideal system, \( K_A = P_A^*/P_T = \text{vapor pressure/total pressure} \).

The relative volatility in multicomponent distillation is always computed with respect to the heavy key
\[ \alpha_j = \frac{K_j}{K_{hk}} \frac{y_{je}}{x_{je}} \frac{x_{je}}{x_{hke}} \] (1)
The value of \( \alpha \) is greater than 1 for any component lighter than the heavy key and less than 1 for any heavier component.
Bubble-point and dew-point calculations

These are required for a flash distillation and for each stage of a multicomponent distillation, calculated by trial-and-error.

For the bubble point,
\[ \sum_{i=1}^{NC} y_i = \sum_{i=1}^{NC} K_i x_i = 1.0 \]  

For the dew point,
\[ \sum_{i=1}^{NC} x_i = \sum_{i=1}^{NC} \frac{y_i}{K_i} = 1.0 \]

First a temperature is assumed and the values of \( K_i \) are obtained from literature. If the summation \( \neq 1 \) choose another temperature.

**Example D9.** Find the bubble-point and the dew-point temperatures and the corresponding vapor and liquid compositions for a mixture of 33 mole % \( n \)-hexane, 37 mole % \( n \)-heptane, and 30 mole % \( n \)-octane at 1.2 atm total pressure.

**Solution.** The logP vs T plot is shown as right. Bubble point. Guess \( T = 105^\circ C \), this is based on the vapor pressure of heptane, the middle component, is 1.2 atm.

<table>
<thead>
<tr>
<th>Component</th>
<th>( P^* ) at 105°C, atm</th>
<th>( K_i = P_i^*/1.2 )</th>
<th>( x_i )</th>
<th>( y_i = K_i x_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Hexane</td>
<td>2.68</td>
<td>2.23</td>
<td>0.33</td>
<td>0.7359</td>
</tr>
<tr>
<td>2. Heptane</td>
<td>1.21</td>
<td>1.01</td>
<td>0.37</td>
<td>0.3737</td>
</tr>
<tr>
<td>3. Octane</td>
<td>0.554</td>
<td>0.462</td>
<td>0.30</td>
<td>0.1386</td>
</tr>
<tr>
<td></td>
<td>( \sum = 1.248 )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Since \( \sum y_i \) is too large, try a lower temperature. Since the major contribution is from the first term, try a temperature where \( K_i \) is lower by a factor of \( 1/1.24 \), 2.68/1.24=2.16. Choose \( T = 96^\circ C \), where \( P_1^* = 2.16 \) atm.

<table>
<thead>
<tr>
<th>Component</th>
<th>( P^* ) at 96°C, atm</th>
<th>( K_i = P_i^*/1.2 )</th>
<th>( x_i )</th>
<th>( K_i x_i )</th>
<th>( y_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Hexane</td>
<td>2.16</td>
<td>1.8</td>
<td>0.33</td>
<td>0.5940</td>
<td>0.604</td>
</tr>
<tr>
<td>2. Heptane</td>
<td>0.93</td>
<td>0.775</td>
<td>0.37</td>
<td>0.2868</td>
<td>0.292</td>
</tr>
<tr>
<td>3. Octane</td>
<td>0.41</td>
<td>0.342</td>
<td>0.30</td>
<td>0.1025</td>
<td>0.104</td>
</tr>
<tr>
<td></td>
<td>( \sum = 0.9833 )</td>
<td></td>
<td></td>
<td>1.000</td>
<td></td>
</tr>
</tbody>
</table>

This is a little bit small. Try another \( T = 97^\circ C \), close enough to 96°C so that \( y_i = \frac{K_i x_i}{\sum_{i=1}^{NC} K_i x_i} \).

Dew point. The dew point is higher than the bubble point, so use 105°C as a first guess.

<table>
<thead>
<tr>
<th>Component</th>
<th>( K_i )</th>
<th>( y_i )</th>
<th>( y_i / K_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Hexane</td>
<td>2.23</td>
<td>0.33</td>
<td>0.148</td>
</tr>
<tr>
<td>2. Heptane</td>
<td>1.01</td>
<td>0.37</td>
<td>0.366</td>
</tr>
<tr>
<td>3. Octane</td>
<td>0.462</td>
<td>0.30</td>
<td>0.649</td>
</tr>
<tr>
<td></td>
<td>( \sum = 1.163 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Since the sum is too high, try a higher temperature, T = 110°C.

<table>
<thead>
<tr>
<th>Component</th>
<th>P_i* at 110°C</th>
<th>K_i = P_i*/1.2</th>
<th>y_i</th>
<th>y_i/K_i</th>
<th>x_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Hexane</td>
<td>3</td>
<td>2.5</td>
<td>0.33</td>
<td>0.132</td>
<td>0.130</td>
</tr>
<tr>
<td>2. Heptane</td>
<td>1.38</td>
<td>1.15</td>
<td>0.37</td>
<td>0.3217</td>
<td>0.317</td>
</tr>
<tr>
<td>3. Octane</td>
<td>0.64</td>
<td>0.533</td>
<td>0.30</td>
<td>0.5625</td>
<td>0.553</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>∑</td>
<td>1.0162</td>
<td>1.000</td>
</tr>
</tbody>
</table>

By extrapolation, the dew point is 110.5°C, and the composition of the liquid in equilibrium with the vapor is calculated by \( x_i = \frac{y_i/K_i}{\sum_{i=1}^{NC} y_i/K_i} \).

Flash distillation

Material balance for component \( i \) gives

\[ y_{Di} = \frac{f - 1}{f} x_{Bi} + \frac{x_{Fi}}{f} \] (4)

where \( f = V/F \) is the fraction of the feed vaporized, \( y_{Di} \) is in equilibrium with \( x_{Bi} \). So \( y_{Di} = K_i x_{Bi} \), substitute this into Eq. (4) and solve for \( x_{Bi} \):

\[ x_{Bi} = \frac{x_{Fi}}{f(K_i - 1) + 1} \] (5)

The summation on \( x_{Bi} \) should be unity.

\[ \sum_{i=1}^{NC} x_{Bi} = 1 = \sum_{i=1}^{NC} \frac{x_{Fi}}{f(K_i - 1) + 1} \] (6)

This equation is again solved by iteration. The final values of \( T \) and \( K_i \) are used to calculate the composition of the top and bottom product streams.

Example D10. The mixture of example D9 is subjected to a flash distillation at 1.2 atm pressure, and 60% of the feed is vaporized.

(a) Find the temperature of the flash and the composition of the liquid and vapor products.

(b) To what temperature must the feed liquid be heated for 60% vaporization on flashing?

Solution.

(a) The flash temperature must lie between the bubble point (97°C) and the dew point (110.5°C).

Assume \( T = 97 + 0.6(110.5 - 97) = 105°C \).

From the figure in example D9, \( K_1 = 2.68/1.2 = 2.23, K_2 = 1.21/1.2 = 1.01, K_3 = 0.554/1.2 = 0.462 \).

The value of \( f \) is 0.6.

The right hand side of Eq. (6) becomes

\[ \frac{0.33}{0.6(2.23 - 1) + 1} + \frac{0.37}{0.6(1.01 - 1) + 1} + \frac{0.30}{0.6(0.462 - 1) + 1} = 0.190 + 0.368 + 0.443 = 1.001 \]

This is close to unity so the flash temperature is 105°C.

The composition of the liquid product is 19% \( n \)-hexane, 36.8% \( n \)-heptane and 44.2% \( n \)-octane.

The composition of the vapor product is computed from the values of \( K \) and \( x \).

<table>
<thead>
<tr>
<th>( n )-hexane, ( y = 0.190(2.23) )</th>
<th>( n )-heptane, ( y = 0.368(1.01) )</th>
<th>( n )-octane, ( y = 0.443(0.462) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \geq 0.424 )</td>
<td>( \geq 0.372 )</td>
<td>( \geq 0.204 )</td>
</tr>
<tr>
<td><strong>(Top)</strong></td>
<td><strong>(Bottom)</strong></td>
<td><strong>(Bottom)</strong></td>
</tr>
<tr>
<td><strong>1.000</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
(b) To determine the temperature of the feed before flashing, an enthalpy balance is made using 105 °C as the reference temperature. The heats of vaporization at 105 °C and the average heat capacities of the liquid from 105 to 200 °C are obtained from the literature.

<table>
<thead>
<tr>
<th>Compound</th>
<th>Cp, cal/(mol K)</th>
<th>ΔHv, cal/mol</th>
</tr>
</thead>
<tbody>
<tr>
<td>n-hexane</td>
<td>62</td>
<td>6370</td>
</tr>
<tr>
<td>n-heptane</td>
<td>70</td>
<td>7510</td>
</tr>
<tr>
<td>n-octane</td>
<td>78</td>
<td>8560</td>
</tr>
</tbody>
</table>

Based on liquid at 105 °C, the enthalpies of the product are

\[ \Delta H_{\text{vapor}} = 0.6(0.424 \times 6370 + 0.372 \times 7510 + 0.204 \times 8560) = 4345 \text{ cal} \]

\[ \Delta H_{\text{liquid}} = 0 \]

For the feed,

\[ \bar{C}_p = 0.33 \times 62 + 0.37 \times 70 + 0.30 \times 78 = 69.8 \text{ cal/(mol °C)} \]

Hence, 69.8(T0 - 105) = 4345

\[ T_0 = 167 \text{ °C} = \text{preheat temperature} \]

- Total Reflux for multicomponent distillation
  (Minimum stages for total reflux)

For binary distillation, if the relative volatility of the two components is approximately constant, we have the following Fenske equation to calculate the minimum number of theoretical steps when a total condenser and a reboiler are used.

\[ N_m = \frac{\ln \left( \frac{x_D - x_B}{1 - x_D} \right)}{\ln \alpha_{AB}} - 1 \]  \hspace{1cm} (7)

Similarly for any two components \((i \& j)\) in multicomponent distillation,

\[ N_m = \frac{\ln \left( \frac{x_{D_i} / x_{B_i}}{x_{D_j} / x_{B_j}} \right)}{\ln \bar{\alpha}_{ij}} - 1 \]  \hspace{1cm} (8)

\[ \bar{\alpha}_{ij} = \frac{3}{\sqrt{\alpha_{Dij} \alpha_{Fij} \alpha_{Bij}}} \]  \hspace{1cm} (9)

The subscripts D, F and B in Eq. (9) refer to the temperatures of the distillate, feed plate, and bottoms in the column. \(N_m\) is the minimum number of equilibrium stages excluding the reboiler, at total reflux.

The Fenske equation is derived as below. At total reflux, \(D = B = 0\), so \(V = L\). Consider the equilibrium partial reboiler for any two components A and B,

\[ \alpha_R = \frac{Y_{AR} / X_{AB}}{Y_{BR} / X_{BB}} \]

Rearrange to obtain
\[
\begin{bmatrix}
    y_A \\
    y_B
\end{bmatrix}_R = \alpha_R \begin{bmatrix}
    x_A \\
    x_B
\end{bmatrix}_B
\] (10)

Material balances for these two components around the reboiler are

\[
V_Ry_A,R = LNxA,N
\] (11a)
\[
V_Ry_B,R = LNxB,N
\] (11b)

Since \( V_R = LN \) for total reflux (\( B = 0 \)), we have

\[
y_A,R = xA,N, \quad y_B,R = xB,N \quad \text{(at total reflux)}
\] (12)

Combining Eqs. (10) and (12),

\[
\begin{bmatrix}
    x_A \\
    x_B
\end{bmatrix}_N = \alpha_R \begin{bmatrix}
    x_A \\
    x_B
\end{bmatrix}_N
\] (13)

If we move up the column to stage \( N \), the equilibrium is

\[
\begin{bmatrix}
    y_A \\
    y_B
\end{bmatrix}_N = \alpha_N \begin{bmatrix}
    x_A \\
    x_B
\end{bmatrix}_N
\] (14)

The mass balances around stage \( N \) simplify to

\[
y_{A,N} = x_{A,N-1} \quad \text{and} \quad y_{B,N} = x_{B,N-1}
\]

Combining these equations to give

\[
\begin{bmatrix}
    x_A \\
    x_B
\end{bmatrix}_{N-1} = \alpha_N \begin{bmatrix}
    x_A \\
    x_B
\end{bmatrix}_N
\] (15)

Eqs. (13) and (15) can be combined to give

\[
\begin{bmatrix}
    x_A \\
    x_b
\end{bmatrix}_{N-1} = \alpha_N \alpha_R \begin{bmatrix}
    x_A \\
    x_B
\end{bmatrix}_B
\] (16)

Which relates the ratio of liquid mole fractions leaving stage \( N-1 \) to the ratio in the reboiler.

Repeating this procedure until we reach the top stage, we have

\[
\begin{bmatrix}
    x_A \\
    x_B
\end{bmatrix}_D = \alpha_1 \alpha_2 \ldots \alpha_{N-1} \alpha_N \alpha_R \begin{bmatrix}
    x_A \\
    x_B
\end{bmatrix}_B
\] (17)

If we define \( \alpha_{AB} \) as the geometric average relative volatility,

\[
\alpha_{AB} = \left[\alpha_1 \alpha_2 \ldots \alpha_{N-1} \alpha_N \alpha_R\right]^{1/(N_m+1)}
\] (18)

Eq. (17) becomes

\[
\begin{bmatrix}
    x_A \\
    x_B
\end{bmatrix}_D = \alpha_{AB}^{N_m+1} \begin{bmatrix}
    x_A \\
    x_B
\end{bmatrix}_B
\] (19)

Solving Eq. (19) for \( N_m \), we have

\[
N_m = \frac{\ln \left[ x_{DA}/x_{DB} \right]}{\ln \alpha_{AB}} - 1
\] (20)

or

\[
N_m = \frac{\ln \left[ D_{x_{DA}}/D_{x_{DB}} \right]}{\ln \alpha_{AB}} - 1
\] (21)

The fractional recovery of A in the distillate is

\[
FR_{AD} = \frac{D_{x_{DA}}}{F_{x_{FA}}}
\] (22)

The fractional recovery of A in the bottoms is

\[
FR_{AB} = \frac{B_{x_{BA}}}{F_{x_{FA}}} = 1 - FR_{AD}
\] (23)

Hence

\[
D_{x_{DA}} = (FR_{AD})(F_{x_{FA}})
\]
\[
B_{x_{BA}} = (1 - FR_{AD})(F_{x_{FA}})
\] (24)

The Fenske equation can also be written as

\[
N_m = \frac{\ln \left[ FR_{AD}/FR_{BD} \right]}{\ln \alpha_{AB}} - 1
\] (26)
Example D11. A mixture with 33% n-hexane, 37% n-heptane, and 30% n-octane is to be distilled to give a distillate product with 0.01 mole fraction n-heptane and a bottoms product with 0.01 mole fraction n-hexane. The column will operate at 1.2 atm. 60% of the feed is vaporized. Calculate the complete product compositions and the minimum number of ideal plates at infinite reflux.

Solution

<table>
<thead>
<tr>
<th>light key (LK)</th>
<th>heavy key (HK)</th>
<th>heavy nonkey (HNK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>n-hexane</td>
<td>n-heptane</td>
<td>n-octane</td>
</tr>
</tbody>
</table>

Assume no n-octane and 0.99 mole fraction n-hexane in the distillate. Take 100 mol/h of feed as the basis

- overall M.B. \( F = D + B = 100 \)
- hexane M.B. \( F x_F = D x_D + B x_B \)

\[100 \times 0.33 = 0.99D + (100-D)(0.01)\]

\[D=32/0.98=32.65 \text{ mol/h}; \ B=100-D=67.35 \text{ mol/h}\]

The composition of the bottoms product can be calculated since this stream contains all the octane, all but 0.01D of the heptane, 0.01 mole fraction of hexane.

<table>
<thead>
<tr>
<th></th>
<th>( F x_F )</th>
<th>( D x_D )</th>
<th>( x_D )</th>
<th>( B x_B )</th>
<th>( x_B )</th>
<th>K at 105°C, 1.2 atm</th>
</tr>
</thead>
<tbody>
<tr>
<td>LK n-C(_6)</td>
<td>33</td>
<td>32.32</td>
<td>0.99</td>
<td>0.68</td>
<td>0.010</td>
<td>2.23</td>
</tr>
<tr>
<td>HK n-C(_7)</td>
<td>37</td>
<td>0.33</td>
<td>0.01</td>
<td>36.67</td>
<td>0.544</td>
<td>1.01</td>
</tr>
<tr>
<td>HNK n-C(_8)</td>
<td>30</td>
<td>0</td>
<td>0</td>
<td>30</td>
<td>0.446</td>
<td>0.462</td>
</tr>
</tbody>
</table>

D=32.65, B=67.35

Use the Fenske equation to obtain the minimum number of plates. The relative volatility of the light key to the heavy key is the ratio of their K factors. The K values at the flash temperature were taken from the previous example.

\[\alpha_{LK,HK} = 2.23/1.01 = 2.21\]

\[\ln\left(\frac{x_{D,LK}}{x_{B,LK}}\right) \quad \ln\left(\frac{0.99/0.01}{0.01/0.544}\right)\]

\[N_m = \frac{\ln \alpha_{LK,HK}}{-1} = \frac{\ln 2.21}{-1} = 10.8 - 1 = 9.8\]

So the minimum number of ideal stages is 9.8 plus a reboiler.

A more accurate estimate of \( N_m \) can be obtained using a mean relative volatility based on the values at the top, middle, and bottom of the column. The top temperature is about 75°C, the boiling point of n-hexane at 1.2 atm, and the relative volatility is 2.53. The bottom temperature is about 115°C by using a bubble point calculation for the bottoms product, giving a relative volatility of 2.15.

\[\alpha_{LK,HK} = \sqrt[2.53]{2.21\times2.15} = 2.29\]

Hence, \( N_m = 9.4\)

To check our assumption of no octane in the distillate, the Fenske equation can be applied to heptane and octane using \( \alpha = K_2/K_3 = 1.01/0.462 = 2.19:\)

\[\ln\left(\frac{0.01/0.544}{x_{D3}/0.446}\right)\]

\[N_m + 1 = 10.4 = \frac{\ln 2.19}{\ln 2.19}\]

from which \( x_{D3} = 2.4\times10^{-6} \), which is negligible.
Minimum Reflux Ratio for multicomponent distillation

At this reflux ratio, the desired separation is just barely possible, but an infinite number of plates is required. The desired separation is usually refers to the amount of light key recovered in the distillate and the amount of heavy key recovered in the bottoms.

Since the only purpose of obtaining the minimum reflux ratio is to estimate the product compositions at $R_{Dm}$ and to ensure that the specified $R_D$ is reasonable, an exact value is not required. There are two approximation methods in estimating $R_{Dm}$:

1. Treating the mixture as a pseudobinary (light & heavy keys)

$$L_{min} = \frac{(D_{x_{D,LK}} / F_{x_{F,LK}}) - \alpha_{LK,HK} (D_{x_{D,HK}} / F_{x_{F,HK}})}{\alpha_{LK,HK} - 1}$$

$$
(27)
$$

2. Underwood method

By using constant average $\alpha$’s and assuming constant molal overflow ($L/V$), the following two equations are solved to obtain $R_{Dm}$:

$$1 - q = \sum \frac{\alpha_i x_{Fi}}{\alpha_i - \phi}$$

$$\frac{V_{min}}{D} = R_{Dm} + 1 = \sum \frac{\alpha_i x_{Di}}{\alpha_i - \phi}$$

where $q$ is the moles of liquid to stripping section of column per mole of feed, $\phi$ is a parameter to be found from Eq. (28). The correct value of $\phi$ should lie between the values of $\alpha$ for the keys.
Example D12. A mixture with 4% n-pentane, 40% n-hexane, 50% n-heptane, and 6% n-octane is to be distilled at 1 atm with 98% of the hexane and 1% of the heptane recovered in the distillate. What is the minimum reflux ratio for a liquid feed at the boiling point?

Solution
The light key is n-hexane and the heavy key is n-heptane. The other components are sufficiently different in volatility to be undistributed. The known information is summarized below (assume 100 moles of feed).

<table>
<thead>
<tr>
<th></th>
<th>x_F</th>
<th>Fx_F</th>
<th>Dx_D</th>
<th>x_D</th>
<th>Bx_B</th>
<th>x_B</th>
</tr>
</thead>
<tbody>
<tr>
<td>n-C_5</td>
<td>0.04</td>
<td>4</td>
<td>4</td>
<td>0.092</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>LK n-C_6</td>
<td>0.40</td>
<td>40</td>
<td>39.2</td>
<td>0.897</td>
<td>0.8</td>
<td>0.014</td>
</tr>
<tr>
<td>HK n-C_7</td>
<td>0.50</td>
<td>50</td>
<td>0.5</td>
<td>0.011</td>
<td>49.5</td>
<td>0.879</td>
</tr>
<tr>
<td>n-C_8</td>
<td>0.06</td>
<td>6</td>
<td>0</td>
<td>0</td>
<td>6</td>
<td>0.107</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>D=43.7</td>
<td></td>
<td>B=56.3</td>
</tr>
</tbody>
</table>

The bubble point of the feed is obtained by checking the equation \( \sum K_{i,F_i} = 1.0 \), this temperature is found to be 80 °C. The corresponding \( \alpha_{LK,HK} \) is 1.39/0.56 = 2.48.

(a) By using the pseudobinary method, we have
\[
\text{L}_{\text{min}} = \frac{39.2 / 40 - 2.48(0.5 / 50)}{2.48 - 1} = 0.645
\]
\[
R_{Dm} = \frac{L_{\text{min}}}{D} = \frac{L_{\text{min}}}{F} = 0.645 \frac{1}{0.437} = 1.48
\]

(b) By using the Underwood method, the K values at 80 °C are converted to relative volatilities and the \( \phi \) value between 1 and 2.48 is found by trial. Since \( q=1 \) (feed at the boiling point), we should have
\[
\sum \frac{\alpha_i x_{Fi}}{\alpha_i - \phi} = 1 - q = 1 - 1 = 0
\]

<table>
<thead>
<tr>
<th>( \alpha_i )</th>
<th>( x_{Fi} )</th>
<th>( \sum \frac{\alpha_i x_{Fi}}{\alpha_i - \phi} ), ( \phi=1.5 )</th>
<th>( \sum \frac{\alpha_i x_{Fi}}{\alpha_i - \phi} ), ( \phi=1.48 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>n-C_5</td>
<td>6.46</td>
<td>0.04</td>
<td>0.052</td>
</tr>
<tr>
<td>n-C_6</td>
<td>2.48</td>
<td>0.40</td>
<td>1.012</td>
</tr>
<tr>
<td>n-C_7</td>
<td>1.0</td>
<td>0.50</td>
<td>-1.00</td>
</tr>
<tr>
<td>n-C_8</td>
<td>0.41</td>
<td>0.06</td>
<td>-0.023</td>
</tr>
</tbody>
</table>

By further trials or interpolation, \( \phi=1.487 \), so
\[
R_{Dm} + 1 = \sum \frac{\alpha_i x_{Di}}{\alpha_i - \phi} = \frac{6.64(0.092)}{1.487} + \frac{2.48(0.897)}{1.487} + \frac{1(0.011)}{1.487}
\]
\[
= 0.119 + 2.24 - 0.023 = 2.336
\]
\[
R_{Dm} = 1.34, \text{ which is } 10\% \text{ less than the value obtained using pseudobinary method.}
\]
Number of ideal plates at operating reflux  
(Gilliland correlation)

Gilliland proposed a simple empirical method to estimate the number of plates in multicomponent distillation. The correlation requires knowledge only of the minimum number of plates at total reflux and the minimum reflux ratio. The correlation is given below.

\[
N = N_{\text{min}} + \frac{R_D - R_{Dm}}{R_D + 1}
\]

\(N\) is the total number of plates in the system, including the reboiler. The same for \(N_{\text{min}}\).

Example D13. Estimate the number of ideal plates required for the separation specified in Example D11, if the reflux ratio is 1.5\(R_{Dm}\).

Solution

From Example D11, the minimum number of ideal stages is 9.4 plus a reboiler, or 10.4. The value of \(R_{Dm}\) is obtained by the Underwood method.

<table>
<thead>
<tr>
<th></th>
<th>(x_F)</th>
<th>(x_D)</th>
<th>(K)</th>
<th>(\alpha)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LK</td>
<td>n-hexane</td>
<td>0.33</td>
<td>0.99</td>
<td>2.23</td>
</tr>
<tr>
<td>HK</td>
<td>n-heptane</td>
<td>0.37</td>
<td>0.01</td>
<td>1.01</td>
</tr>
<tr>
<td>n-octane</td>
<td>0.30</td>
<td>0.0</td>
<td>0.462</td>
<td>0.457</td>
</tr>
</tbody>
</table>

For a saturated liquid feed, \(q=1\),

\[
\sum i \frac{\alpha_i x_{Fi}}{\alpha_i - \phi} = 1 - q = 1 - 1 = 0
\]

By trial, \(\phi=1.45\),

\[
R_{Dm} + 1 = \sum i \frac{\alpha_i x_{Di}}{\alpha_i - \phi} = \frac{2.21(0.99)}{2.21 - 1.45} + \frac{1.0(0.01)}{1 - 1.45} = 2.86
\]

\[
R_{Dm} = 1.86, \quad R_D = 1.5 \times 1.86 = 2.79
\]

\[
\frac{R_D - R_{Dm}}{R_D + 1} = \frac{2.79 - 1.86}{2.79 + 1} = 0.245
\]

From the Gilliland correlation,

\[
\frac{N - N_{\text{min}}}{N + 1} = 0.41
\]

\[
N - 10.4 = 0.41N + 0.41
\]

\[
N = 10.81/0.59 = 18.3 \text{ stages, including the reboiler}
\]